

Calculus II: Problem Set #1 (Winter 2010)

1. For each problem below, decide whether it can be solved using integration by substitution? If it **can**, then state the u and du that work, and rewrite the integral in terms of u and du (then stop). If the problem **cannot** be solved by u -substitution, then explain what choice(s) of u you might have tried, and why they wouldn't work. [10 points]

a. $\int \frac{dx}{(4+9x)^2}$ b. $\int \frac{dx}{4+9x^2}$ c. $\int \frac{xdx}{4+9x^2}$ d. $\int \frac{e^x dx}{4+9e^{2x}}$

2. There are two pairs of problems below that are exactly the same. You won't see why until you do the integration, showing all the steps. Find the pairs and then explain how the matched integrals are fundamentally the same. [10 points]

a. $\int_1^e \frac{\ln x dx}{x}$ b. $\int_0^{\ln 2} \frac{e^x dx}{1+e^x}$ c. $\int_0^1 (x+2)^{-1} dx$ d. $\int_0^{\pi/2} \sin x \cos x dx$

3. [20 points] Print the page of integral "cards" on the next page and then cut them out so that each problem is separated from the others. Use Wolfram Alpha (www.wolframalpha.com) to solve each integral and write down the answer in the space provided. Then rearrange the cards into groups that seem to have similar kinds of structures in their problems and answers. Once you've got them grouped, explain what relates the problems. Then write up your findings. You might write statements something like this ... "When the integrals are like this: _____, the solutions seem to be similar in that _____. Then provide problems and answers as support for your conjectures. You should not attempt to completely figure out how to do the problems (for now), I just want you to look for patterns and ways to discriminate that certain types of problems give certain types of answers.

For example, here's how to do the first one:

[http://www.wolframalpha.com/input/?i=integrate+10/\(x^2-16\)](http://www.wolframalpha.com/input/?i=integrate+10/(x^2-16))

WolframAlpha™ computational knowledge engine

integrate 10/(x²-16)

Indefinite integral: $\int \frac{10}{x^2-16} dx = 10 \left(\frac{1}{8} \log(4-x) - \frac{1}{8} \log(x+4) \right) + \text{constant}$

Show steps

log(x) is the natural logarithm »

You will want to clean up the answer slightly when you write it down as this will make it easier to see patterns later. Note that Wolfram Alpha uses $\log(x)$ when it means natural log.

1	$\int \frac{10}{x^2 - 16} dx$	$\frac{5}{4} \ln(4 - x) - \frac{5}{4} \ln(x + 4) + C$
2	$\int \frac{10}{x^2 + 16} dx$	
3	$\int \frac{10x}{x^2 + 16} dx$	
4	$\int \frac{10x + 10}{x^2 + 16} dx$	
5	$\int \frac{10x + 1}{x^2 + 8x + 16} dx$	
6	$\int \frac{10}{x^2 + 8x + 16} dx$	
7	$\int \frac{10}{x^2 + 6x + 12} dx$	
8	$\int \frac{10}{x^2 + 10x + 25} dx$	
9	$\int \frac{10x + 10}{x^2 - 10x + 25} dx$	
10	$\int \frac{4x - 20}{x^2 - 10x + 25} dx$	

11	$\int \frac{10}{x^2 + 25} dx$	
12	$\int \frac{10}{x^2 + 4x + 5} dx$	
13	$\int \frac{10}{2x^2 + 12x + 18} dx$	
14	$\int \frac{10x + 10}{2x^2 + 12x + 18} dx$	
15	$\int \frac{x + 1}{x^2 + 2x + 5} dx$	
16	$\int \frac{10}{x^2 - 4x + 5} dx$	
17	$\int \frac{10x}{x^2 + 25} dx$	
18	$\int \frac{10}{x^2 - 3x - 18} dx$	
19	$\int \frac{10x + 1}{x^2 - 3x - 18} dx$	
20	$\int \frac{10x}{x^2 + 10x + 16} dx$	